can not be taken as unambiguous evidence against the complex formation hypothesis.

For a good fuel the requirements are a) high density; b) low viscosity; and c) low surface tension.[8] Comparing the results for hydrazine and mixed fuel given in Table 1, it can be seen that mixed fuel is no better with respect to a and b, however, it is a better choice from the viewpoint of c.

Data recorded in Table 3 show that the ignition delay is longer in the case of mixed fuel as compared to hydrazine. This is expected since ethyl alcohol partly inhibits the decomposition of hydrazine. It is also observed that ignition delay increases with the increase of ethyl alcohol concentration. However, the delay can be reduced by the addition of hydrazinium nitrate in the fuel. Thus, for 82 weight percent hydrazine it can be reduced to 30 msec on adding hydrazinium nitrate.

The qualitative analysis of gaseous combustion products indicated the presence of N_2O, N_3H, NH_3 and N_2. When the mixed fuel is used the only additional product is aldehyde produced by the oxidation of alcohol. This means that during the combustion of mixed fuels simultaneous reactions take place.

The values of performance parameters recorded in Table 4 show that I_{sp}, Tc, and c^* all decrease with decrease in hydrazine content. The storability of the mixed fuel increases with the decrease in the concentration of hydrazine. For practical purposes, optimization would be necessary and judged from all angles mixed fuel containing 70–80%, by weight of hydrazine should be good enough. Table 5 shows the variation of specific impulse with mixture ratio for a typical composition of the mixed fuel. The results show that mixture ratio = 1.5 is the best.

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Average Structural Response to Locally Stationary Random Excitation

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Introduction

IN physical problems, the concept of a locally stationary process can be used to approximate nonstationary excitations when the average instantaneous power is varying slowly with respect to memory time.[1] Using this concept, it is sometimes possible to break up a nonstationary excitation into a number of smaller samples that exhibit weakly stationary properties.

The response of a time-invariant linear system to this type of nonstationary excitation follows from well known principles. Analogous, in the probabilistic sense, to the transient (nonstationary) and steady-state (weakly stationary) response to a stationary excitation,[2] there is a transient and average (weakly stationary) response to a nonstationary excitation.[3] The transient response of a time-invariant linear system to a locally stationary excitation is discussed in Ref. 4. Of interest here is the average response.

It is noted that the average response to a locally stationary excitation is of interest because it plays an important role in some design studies. In aeronautical applications, for example, this concept is used to approximate the nonstationary atmospheric turbulence phenomenon, and military design specifications for aircraft have been developed using this approach.

In the present Note, the average properties of the excitation are determined by applying a time version of the usual treatment of a sequence of random variables. This approach corresponds to the singular case considered in Ref. 3 and appears to be alluded to in Ref. 5. For a stationary process, this technique leads to the usual theorems on ergodicity. The average response properties follow by considering the average excitation as a weakly stationary input.

General Concepts

In the present context, locally stationary means that a long record $x(t)$ of length T can be separated into a sequence of distinct independent phases ${}^jx(t), j = 1,\ldots,N$, where each phase lasts for a time ΔT_j which is long enough to exhibit dominantly weakly stationary properties. That is

$$x(t) = {}^jx(t) \quad \text{on} \quad (T_{j-1}, T_j], \qquad j = 1,\ldots,N \tag{1}$$

where

$$T = \sum_{j=1}^{N} \Delta T_j \tag{2}$$

and

$$\Delta T_j = T_j - T_{j-1}, \qquad j = 1,\ldots,N \tag{3}$$

If each phase is described by its mean ${}^j\mu_x$, its autocorrelation ${}^jR_x(\tau)$, and its distribution ${}^jF_x(x)$, then these quantities are given by the expected values

$${}^j\mu_x = E[{}^jx(t)] \tag{4}$$

$${}^jR_x(\tau) = E[{}^jx(t+\tau){}^jx(t)] \tag{5}$$

$${}^jF_x(x) = E[{}^jz(t)] \tag{6}$$

where ${}^jz(t)$ is the process

$${}^jz(t) = \begin{cases} 1, & \text{if } {}^jx(t) \leqq x \\ 0, & \text{if } {}^jx(t) > x \end{cases} \tag{7}$$

and x is an arbitrary but fixed constant.

One approach to determining the average statistical properties of $x(t)$ is to formulate the over-all properties of the sequence of time dependent random variables ${}^jx(t), j = 1,\ldots,N$. To illustrate, consider the time average

$$\bar{x}(t) = \frac{1}{T}\int_0^T x(t)\,dt \tag{8}$$

of the given process $x(t)$. Clearly, from Eq. (1),

$$\bar{x}(t) = \frac{1}{T}\sum_{j=1}^{N}\int_{T_{j-1}}^{T_j} {}^jx(t)\,dt \tag{9}$$

where $\bar{x}(t)$ is a random variable.

Equation (9) is a time version of the estimate of the sample mean for a sequence of random variables. Therefore, the over-all mean is the expected value of Eq. (9). That is,

$$\overline{E[x(t)]} = \frac{1}{T}\sum_{j=1}^{N}\int_{T_{j-1}}^{T_j} E[{}^jx(t)]\,dt \tag{10}$$

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where the order of the expected value and the time average are interchanged.

Equation (10) is the most likely or average mean value of $x(t)$. Noting that the integrand in this equation is constant for a locally stationary process, it follows from Eqs. (3) and (4) that

$$\bar{\mu}_x = \frac{1}{T}\sum_{j=1}^{N} \Delta T_j\, {}^j\mu_x \tag{11}$$

where $\bar{\mu}_x = \overline{E[x(t)]}$. The average value given by Eq. (11) is time-invariant and thus has the properties of a stationary process. The contributions of the phases ${}^j\mu_x$ are weighted according to their relative times of occurrence $\Delta T_j/T$.

The average autocorrelation and distribution follow in much the same way. These quantities are also time-invariant weighted average values and are obtained by taking expected values of time averages that represent sample estimates.

The average autocorrelation is given by

$$\overline{E[x(t+\tau)x(t)]} = \frac{1}{T-\tau}\sum_{j=1}^{N}\int_{T_{j-1}}^{T_j-\tau} E[{}^jx(t+\tau)\,{}^jx(t)]\,dt \tag{12}$$

where $\overline{E[x(t+\tau)x(t)]}$ is the expected value of the sample estimate of the autocorrelation for a sequence of time dependent random variables. Letting $\bar{R}_x(\tau) = \overline{E[x(t+\tau)x(t)]}$ and using Eqs. (3) and (5) in Eq. (12) gives the weighted average value

$$\bar{R}_x(\tau) = \frac{1}{T}\sum_{j=1}^{N} \Delta T_j\, {}^jR_x(\tau) \tag{13}$$

where ${}^jR_x(\tau) = 0$ for $\tau > \Delta T_j$ and $T \gg \tau$.

The average distribution is obtained from

$$\overline{E[z(t)]} = \frac{1}{T}\sum_{j=1}^{N}\int_{T_{j-1}}^{T_j} E[{}^jz(t)]\,dt \tag{14}$$

where $\overline{E[z(t)]}$ is the expected value of the average time that $x(t)$ remains below the line $x(t) = x$. Using Eqs. (3) and (6) in Eq. (14) gives the weighted average value

$$\bar{F}_x(x) = \frac{1}{T}\sum_{j=1}^{N} \Delta T_j\, {}^jF_x(x) \tag{15}$$

where $\bar{F}_x(x) = \overline{E[z(t)]}$.

Since the phases ${}^jx(t)$, $j = 1,\ldots,N$, are assumed weakly stationary, Eqs. (11, 13, and 15) completely describe the given process $x(t)$. Additional properties such as the mean square, variance, and power-spectrum can be derived from these equations and will also be time-invariant weighted average values. The average variance is given by

$$\bar{\sigma}_x{}^2 = \bar{R}_x(0) - \bar{\mu}_x{}^2 \tag{16}$$

where $\bar{R}_x(0)$ is the average mean square value obtained by setting $\tau = 0$ in Eq. (13). The average power-spectrum and the average autocorrelation are the Fourier transform pair

$$\bar{S}_x(\omega) = \int_{-\infty}^{\infty} \bar{R}_x(\tau)\, e^{-i\omega\tau}\, d\tau \tag{17}$$

$$\bar{R}_x(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \bar{S}_x(\omega)\, e^{i\omega\tau}\, d\omega \tag{18}$$

where $\bar{S}_x(\omega)$ denotes the average power-spectrum.

Response

Suppose that the given process $x(t)$, described by Eqs. (1–7), is the excitation (input) to a time-invariant linear system. The corresponding response (output) can be expressed by

$$y(t) = \int_{-\infty}^{\infty} h(\theta)x(t-\theta)\, d\theta \tag{19}$$

where $h(t)$ is the system's impulsive response function.

Since the average excitation properties are characteristic of a weakly stationary process, the corresponding average response properties are also characteristic of a weakly stationary process. That is, the average response properties can be obtained by the usual stationary relationships, provided that the average excitation properties are used to represent the input process [see Eqs. (20–26)]. This can be shown by taking the expected values of the appropriate time averages (sample estimates) for the process $y(t)$, and leads to the conclusion that the average response properties are also time-invariant weighted average values.

Thus, the average mean $\bar{\mu}_y$, the average autocorrelation $\bar{R}_y(\tau)$, and the average distribution $\bar{F}_y(y)$ completely describe the process $y(t)$ and are given by

$$\bar{\mu}_y = H(0)\bar{\mu}_x \tag{20}$$

$$\bar{R}_y(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} |H(\omega)|^2 \bar{S}_x(\omega)\, e^{i\omega\tau}\, d\omega \tag{21}$$

$$\bar{F}_y(y) = \frac{1}{T}\sum_{j=1}^{N} \Delta T_j\, {}^jF_y(y) \tag{22}$$

where $H(\omega)$ is the system's complex frequency response function. In Eq. (22), ${}^jF_y(y)$ represents the output distribution that corresponds to the $x(t)$ phase. Unfortunately, it is not generally possible to express ${}^jF_y(y)$ in terms of ${}^jF_x(x)$ unless the phases are considered locally gaussian.

The average mean square $\bar{\sigma}_y{}^2$ and the average power-spectrum $\bar{S}_y(\omega)$ of the response are given by

$$\bar{\sigma}_y{}^2 = \bar{R}_y(0) - \bar{\mu}_y{}^2 \tag{23}$$

$$\bar{S}_y(\omega) = |H(\omega)|^2 \bar{S}_x(\omega) \tag{24}$$

where $\bar{R}_y(0)$ is the average mean square obtained by setting $\tau = 0$ in Eq. (21). Clearly, $\bar{S}_y(\omega)$ and $\bar{R}_y(\tau)$ are the Fourier transform pair

$$\bar{S}_y(\omega) = \int_{-\infty}^{\infty} \bar{R}_y(\omega)\, e^{-i\omega\tau}\, d\tau \tag{25}$$

$$\bar{R}_y(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \bar{S}_x(\omega)\, e^{i\omega\tau}\, d\omega \tag{26}$$

Closure

In some applications, the excitation $x(t)$ represents the intermittent or repeated application of several distinct independent phenomena—say $x_1(t)$ and $x_2(t)$ for simplicity. For this case it is sometimes possible to approximate $x(t)$ by choosing locally stationary phases such that

$$x(t) = \begin{cases} {}^ix_1(t) \text{ on } (T_{i-1}, T_i], & i = 1,\ldots,l \\ {}^jx_2(t) \text{ on } (T_{j-1}, T_j], & j = 1,\ldots,m \\ {}^k[x_1(t)+x_2(t)] \text{ on } (T_{k-1}, T_k], & k = 1,\ldots,n \end{cases} \tag{27}$$

where

$$N = l+m+n, \quad T = \sum_{i=1}^{l} \Delta T_i + \sum_{j=1}^{m} \Delta T_j + \sum_{k=1}^{n} \Delta T_k$$

and

$$\Delta T_i = T_i - T_{i-1}, \quad \Delta T_j = T_j - T_{j-1}, \quad \Delta T_k = T_k - T_{k-1}$$

It can be shown that in this case, Eq. (11) can be expressed in the form

$$\bar{\mu}_x = p_1\, {}^i\bar{\mu}_{x_1} + p_2\, {}^j\bar{\mu}_{x_2} + p_{1,2}({}^k\bar{\mu}_{x_1} + {}^k\bar{\mu}_{x_2}) \tag{28}$$

where

$${}^i\bar{\mu}_{x_1} = \sum_{i=1}^{l} \Delta T_i\, E[{}^ix_1(t)] \Big/ \sum_{i=1}^{l} \Delta T_i,$$

$${}^k\bar{\mu}_{x_1} = \sum_{k=1}^{n} \Delta T_k\, E[{}^kx_1(t)] \Big/ \sum_{k=1}^{n} \Delta T_k$$

$${}^j\bar{\mu}_{x_2} = \sum_{j=1}^{m} \Delta T_j\, E[{}^jx_2(t)] \Big/ \sum_{j=1}^{m} \Delta T_j,$$

$${}^k\bar{\mu}_{x_2} = \sum_{k=1}^{n} \Delta T_k\, E[{}^kx_2(t)] \Big/ \sum_{k=1}^{n} \Delta T_k$$

and where the time fractions

$$p_1 = \frac{1}{T}\sum_{i=1}^{l} \Delta T_i, \quad p_2 = \frac{1}{T}\sum_{j=1}^{m} \Delta T_j$$

and

$$p_{1,2} = \frac{1}{T}\sum_{k=1}^{n} \Delta T_k$$

are the relative frequencies or probabilities of occurrence of $x_1(t)$ alone, $x_2(t)$ alone, and $x_1(t)+x_2(t)$, respectively. Clearly, $1 = p_1+p_2+p_{1,2}$ is the probability of an event certain to occur.

In terms of cumulative frequencies or probabilities of occurrence, Eq. (28) reduces to

$$\bar{\mu}_x = \hat{p}_1 \bar{\mu}_{x_1} + \hat{p}_2 \bar{\mu}_{x_2} \tag{29}$$

where

$$\bar{\mu}_{x_1} = (p_1{}^i \bar{\mu}_{x_1} + p_{1,2}{}^k \bar{\mu}_{x_1})/\hat{p}_1, \quad \bar{\mu}_{x_2} = (p_2{}^j \bar{\mu}_{x_2} + p_{1,2}{}^k \bar{\mu}_{x_2})/\hat{p}_2$$

and where the time fractions $\hat{p}_1 = p_1+p_{1,2}$ and $\hat{p}_2 = p_2+p_{1,2}$ are the probabilities of occurrence of $x_1(t)$ or $x_1(t)+x_2(t)$ and $x_2(t)$ or $x_1(t)+x_2(t)$, respectively.

Further development of this case is left to the interested reader. It is important to note, however, that setting $p_{1,2} = 0$ implies that $x_1(t)$ and $x_2(t)$ are mutually exclusive 100% of the time, and setting $p_{1,2} = 1$ implies $x_1(t)$ and $x_2(t)$ act concurrently 100% of the time. The former case illustrates how to resolve $x(t)$ into mutually exclusive components, and the latter case leads to the principle of linear superposition. Finally, it should be noted that the p and $\hat{p}$ values are discrete and can also be determined by counting occurrences.

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On the Accuracy of the Taylor Approximation for Structure Resizing

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Introduction

CONDUCTING full analyses each time a structure is modified (i.e., structural elements resized, material properties changed) makes the repetitive design process costly. Thus, several "short cut" structural modification algorithms[1-3] have been proposed. Their common drawback is that it becomes more economical to reanalyze the complete new structure rather than to use the modification method even when only a relatively small number of elements are modified. However, use of the Taylor expansion[4] for structural resizing is applicable to modifications involving a large number of structural members, ultimately even resizing all elements of a structure simultaneously. Since it is an approximate method, the potential it offers hinges on its accuracy. Therefore, results of a series of numerical experiments to test the method's accuracy were obtained for a large complex structure, the midsection of an idealized aircraft fuselage (Fig. 1). A more detailed description of the method and its application is found in Ref. 5.

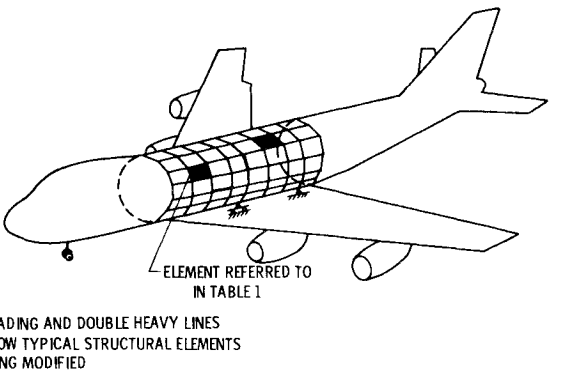

Fig. 1 Fuselage midsection.

Method

The pertinent equations are the familiar load $\{L\}$, deflection $\{u\}$, equation

$$[K]\{u\} = \{L\} \tag{1}$$

its derivative with respect to the design variable V_i

$$[K](\partial\{u\}/\partial V_i) = -(\partial[K]/\partial V_i)\{u\} \tag{2}$$

and the first-order Taylor expansion for the displacements

$$\{u\}_{\text{modified}} = \{u\}_{\text{original}} + (\partial\{u\}/\partial V_i)\cdot\Delta V_i \tag{3}$$

Note that the stiffness matrix, $[K]$, is a function of the design variables, V_i (i.e., thickness, area, or any stiffness property of individual finite element), however, $\{L\}$ is considered independent of V_i thus neglecting dead load.

Considering the formal similarity of Eqs. (1) and (2), one may obtain gradients, $\partial\{u\}/\partial V_i$ by repeating only the back substitution (at 1/39th the cost of reanalysis in this case) portion of the original solution of Eq. (1) with the right-hand side of Eq. (2) playing the role of a pseudo load.

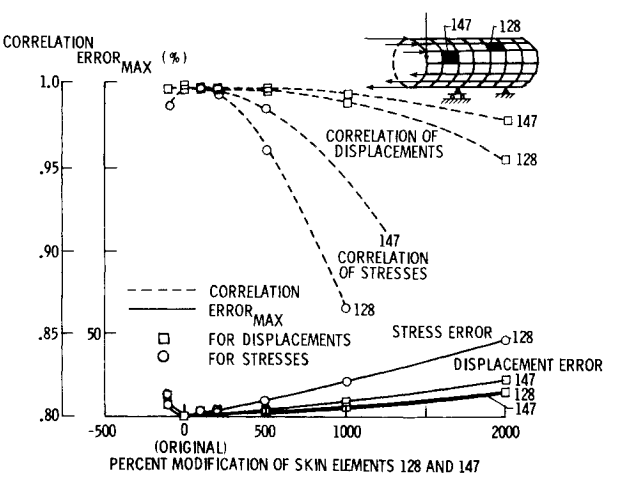

Fig. 2 Accuracy of Taylor expansion for resized skin elements.

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